Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Thursday 5 November 2020

Morning (Time: 2 hours 30 minutes) | Paper Reference **4MB1/02R**

Mathematics B
Paper 2R

You must have:
Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used**.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.

Turn over ▶

P62019A

1/1/1/1/

P 6 2 0 1 9 A 0 1 3 6

Pearson

Answer all TWELVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The manufacturer's price for a *Jinko* car is $\$x$

Ben was given a 7% discount on the manufacturer's price when he bought a *Jinko*.
Ben paid \$23 622 when he bought his *Jinko*.

(a) Calculate the value of x.

(2)

After a year Ben sold his *Jinko* for \$19 880

(b) Calculate the percentage loss, to 3 significant figures, on the price Ben paid for his *Jinko*.

(2)

During the year that Ben owned the *Jinko*, he travelled d km in the car.
The average fuel consumption of the car was 10 km per litre.
The average cost of the fuel he used was \$1.40 per litre.
Other costs for the car in the year came to \$938

The cost per km, including the loss in value, of his *Jinko* to Ben during the year that he owned the car was \$0.40

(c) Calculate the value of d.

(4)

(a) Here you must use the multiplier and work backwards.

Since the full price was 100%, 7% off the full price means the sale percentage will be 93%.

$x \times 0.93 = 23\ 622$

$x = 25400$

(b) Percentage Loss = $\dfrac{\text{Change}}{\text{Original}} \times 100$

$\dfrac{23\ 622 - 19880}{23\ 622} \times 100 = 15.8\%$

P 6 2 0 1 9 A 0 2 3 6

Question 1 continued

(c) Here you need to work out the distance in km Ben travelled in the car from the cost of fuel and the total spent on the car.

First find an expression for the total cost of fuel for d km. Since the fuel cost is per 10km, you will need to divide d by 10.

$$\frac{d}{10} \times 1.40 = 0.14d$$

Next find the total cost for the car including the loss in value in terms of d.

$23622 - 19\,880 = 3742$

$0.14d + 3742 + 938 = 0.4 \times d$

Bring all terms with d to one side and solve.

$0.14d + 3742 + 938 = 0.4d$

$0.14d + 4680 = 0.4d$ -0.14d

-0.14d $4680 = 0.26d$

÷0.14 $18000 = d$ ÷0.14

(Total for Question 1 is 8 marks)

P 6 2 0 1 9 A 0 3 3 6

2 (a) Find the Highest Common Factor (HCF) of 75, 90 and 120

(2)

Bhu sets the alarm on her phone to sound at 09 10
Her alarm then sounds every 12 minutes.

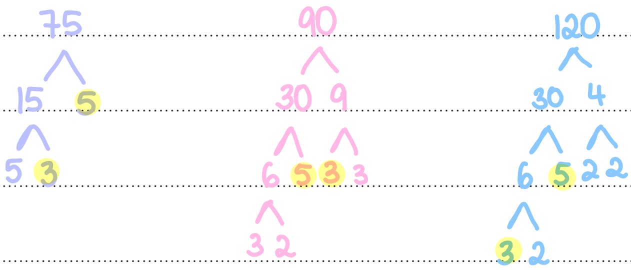
Dax sets the alarm on his phone to sound at 09 30
His alarm then sounds every 8 minutes.

Bhu's alarm sounds at 09 10 and Dax's alarm sounds at 09 30

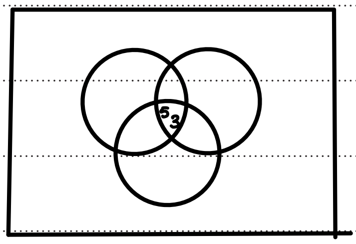
(b) Find the first time after 09 30 that both alarms will sound at the same time.

(2)

(a) Form factor trees for 75, 90 and 120



Draw a Venn diagram and find the HCF using the intersection.



$5 \times 3 = \underline{\underline{15}}$

(b) The easiest way to do this would be to add on 8min or 12min from 09:30.

Bhu		Dax	
09 : 10	+8min	09 : 30	+ 12min
09 : 22	+8min	09 : 38	+ 12min
09 : 34	+8min	09 : 46	
09 : 46			

∴ $\underline{\underline{09:46}}$

P 6 2 0 1 9 A 0 4 3 6

Question 2 continued

(Total for Question 2 is 4 marks)

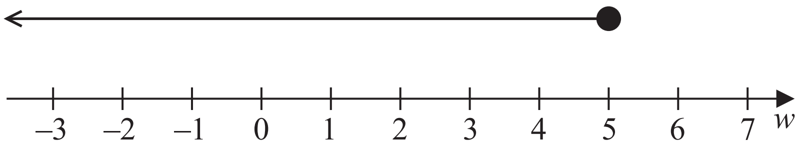
3 (a) Solve the equation $3a + 6 = 4 - 5a$

(2)

(b) Solve the inequality $3p > 6p + 12$

(2)

An inequality, in w, is shown on a number line below.



(c) Write down the inequality.

(1)

(d) Write down the 3 inequalities that define the shaded region R in Figure 1

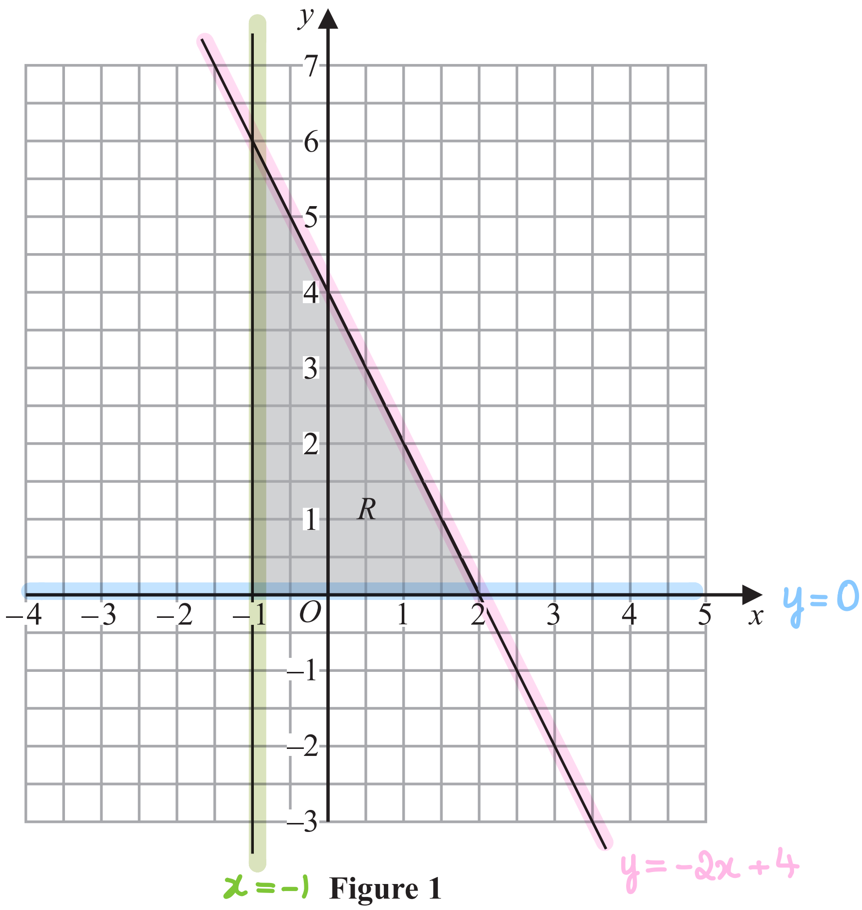


Figure 1

(3)

(a) $3a + 6 = 4 - 5a$

$+5a$ $\quad$ $8a + 6 = 4$ $\quad$ $+5a$

-6 $\quad$ $8a = -2$ $\quad$ -6

$\div 8$ $\quad$ $a = -\frac{1}{4}$ $\quad$ $\div 8$

Question 3 continued

(b)

$$3p > 6p + 12$$

$$\xrightarrow{-6p} -3p > 12 \xleftarrow{-6p}$$

$$\xrightarrow{\div 3} -p > 4 \xleftarrow{\div 3}$$

$$\xrightarrow{\div -1} p < -4 \xleftarrow{\div -1}$$

↳ You need to flip the sign when you divide by -1.

(c) The arrow points left so w is less than 5. The circle is filled up so is less than or equal.

$\underline{\underline{w \leqslant 5}}$

(d) The shaded region is to the right of the vertical line $x = -1$ so x is greater than -1 and the line is solid so greater than or equal to.

$x \geqslant -1$

The shaded region is above the line $y = 0$ so y is greater than 0 and the line is solid so is greater than or equal.

$y \geqslant 0$

The line shaded pink has gradient -2 since for every one x, you move down 2 steps for y. And, it has y-intercept 4 since it crosses the y-axis at $(0,4)$. Since the shaded region is below the line it is less than y and the line is solid it is less than or equal to.

$y \leqslant -2x + 4$

(Total for Question 3 is 8 marks)

4 A youth club has introduced three new activities

badminton (B), cookery (C) and drama (D).

Each of the 75 members of the youth club is asked to say in which of these activities they have participated.

Their answers showed that of the 75 members

all have participated in at least one of these activities

27 have participated in badminton and drama

31 have participated in badminton and cookery

23 have participated in cookery and drama

48 have participated in badminton

49 have participated in cookery

40 have participated in drama.

Let x be the number of members of the youth club who have participated in all three activities.

(a) Using all this information, complete the Venn diagram opposite to show, in terms of x, the number of elements in each appropriate subset.

(3)

(b) Find the value of x.

(2)

(c) Find

(i) $n(B \cap C')$

(ii) $n\left(\left[(B \cup C) \cap D\right]'\right)$

(2)

One of the members of the youth club is picked at random.

Given that this member has participated in cooking,

(d) find the probability that this member has not participated in any other activity.

(2)

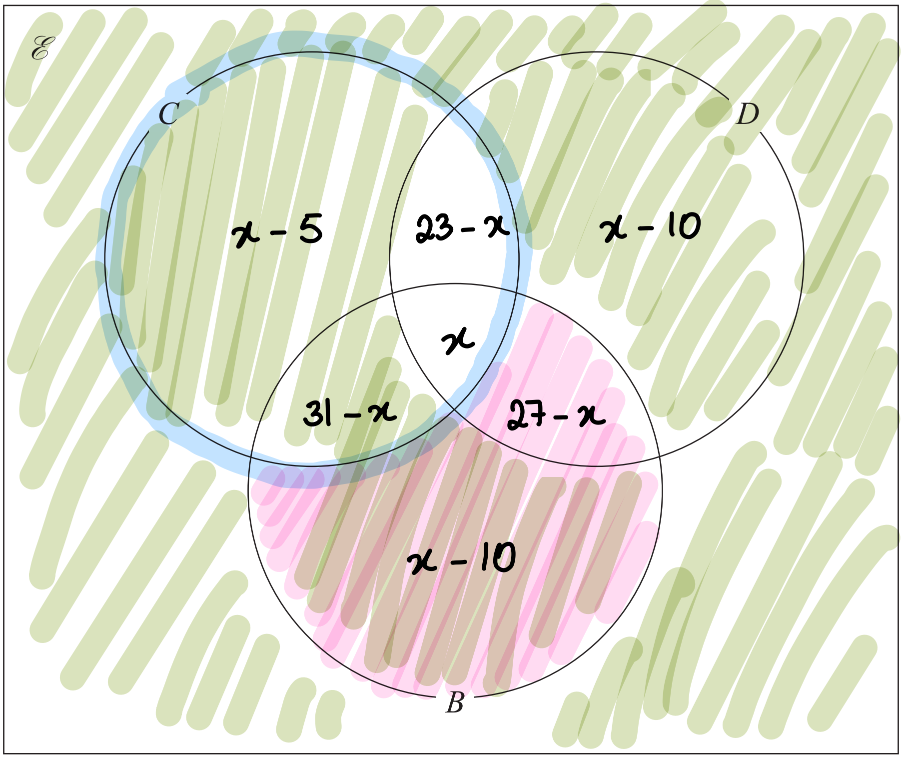
(a) Subtract x from the number of members who play two sports and subtract the intersections from the number of members who play the sport.

$48-(x)-(31-x)-(27-x)=x-10$

$49-(x)-(31-x)-(23-x)=x-5$

$40-(x)-(27-x)-(23-x)=x-10$

Question 4 continued



(b) Since the total number of members is 75, you can add all values in the Venn diagram and make equal to 75 to find x.

$(x-5)+(23-x)+(x-10)+(31-x)+(x)+(27-x)+(x-10)=75$

$x+56=75$

$x=19$

(c)(i) n(B ∩ C′) means the number of members who participate in basketball but not in cookery.

$[27-(19)]+[(19)-10]=17$

(ii) $n([(B \cup C) \cap D]')$ means the number of participants who are note in the intersection of B or C with D.

$[(19)-5]+[(19)-10]+[31-(19)]+[(19)-10]=44$

Turn over for a spare Venn diagram if you need to redraw your diagram.

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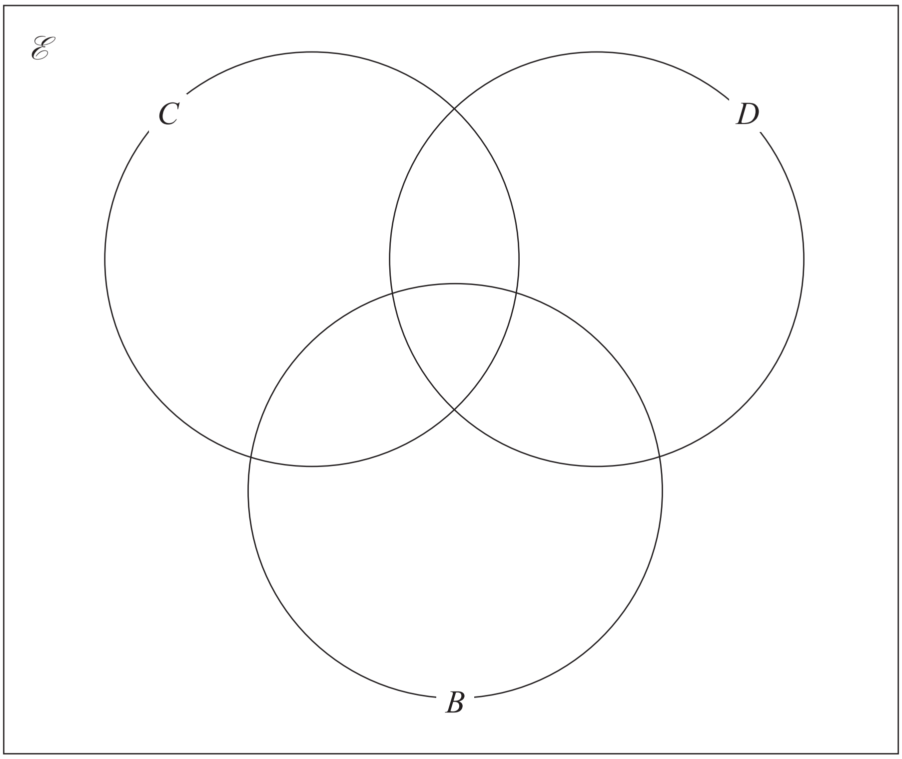
Question 4 continued

Only use this diagram if you need to redraw your Venn diagram.



(a) Here you need to find the probability as a fraction.

We are given that the total number of people in cookery are 49.

The number of people in only cookery are (19) − 5 = 14

Probability = $\frac{14}{49}$

(Total for Question 4 is 9 marks)

5 (a) Factorise $x^2 + 5x - 6$

(2)

(b) Express as a single fraction in its simplest form

$$\frac{x+3}{5} - \frac{2x-2}{4}$$

(3)

(a) You can factorise x^2+5x-6 by finding out what two numbers add to make 5 and multiply to make -6.

These are 6 and -1.

$x^2+5x-6 = (x+6)(x-1)$

(b) You can subtract fractions when you have the same denominator.

$$\frac{4(x+3)}{(4)(5)} - \frac{5(2x-2)}{(5)(4)} = \frac{(4x+12)-(10x-10)}{20}$$

$$= \frac{22-6x}{20}$$

$$= \frac{\cancel{2}(11-3x)}{\cancel{2}(10)}$$

Take out a factor of 2 and cancel out

(Total for Question 5 is 5 marks)

6 The points with coordinates (1, 1), (3, 1) and (4, 3) are the vertices of triangle A.

(a) On the grid opposite, draw and label triangle A.

(1)

Triangle B is the image of triangle A under the transformation with matrix **M** where

$$\mathbf{M} = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

(b) On the grid opposite, draw and label triangle B.

(3)

Triangle C is the image of triangle B under a reflection in the line with equation $y = 0$

(c) On the grid opposite, draw and label triangle C.

(1)

Triangle A is transformed to triangle C under the transformation with matrix **N**.

(d) Find matrix **N**.

(2)

Triangle D is the image of triangle C under a rotation through 180º about the point (1, 1)

(e) On the grid opposite, draw and label triangle D.

(2)

(f) Describe fully the **single** transformation that maps triangle D onto triangle A.

(3)

(a) On grid

(b) You must transform each point on triangle A to triangle B by multiplying by the matrix M.

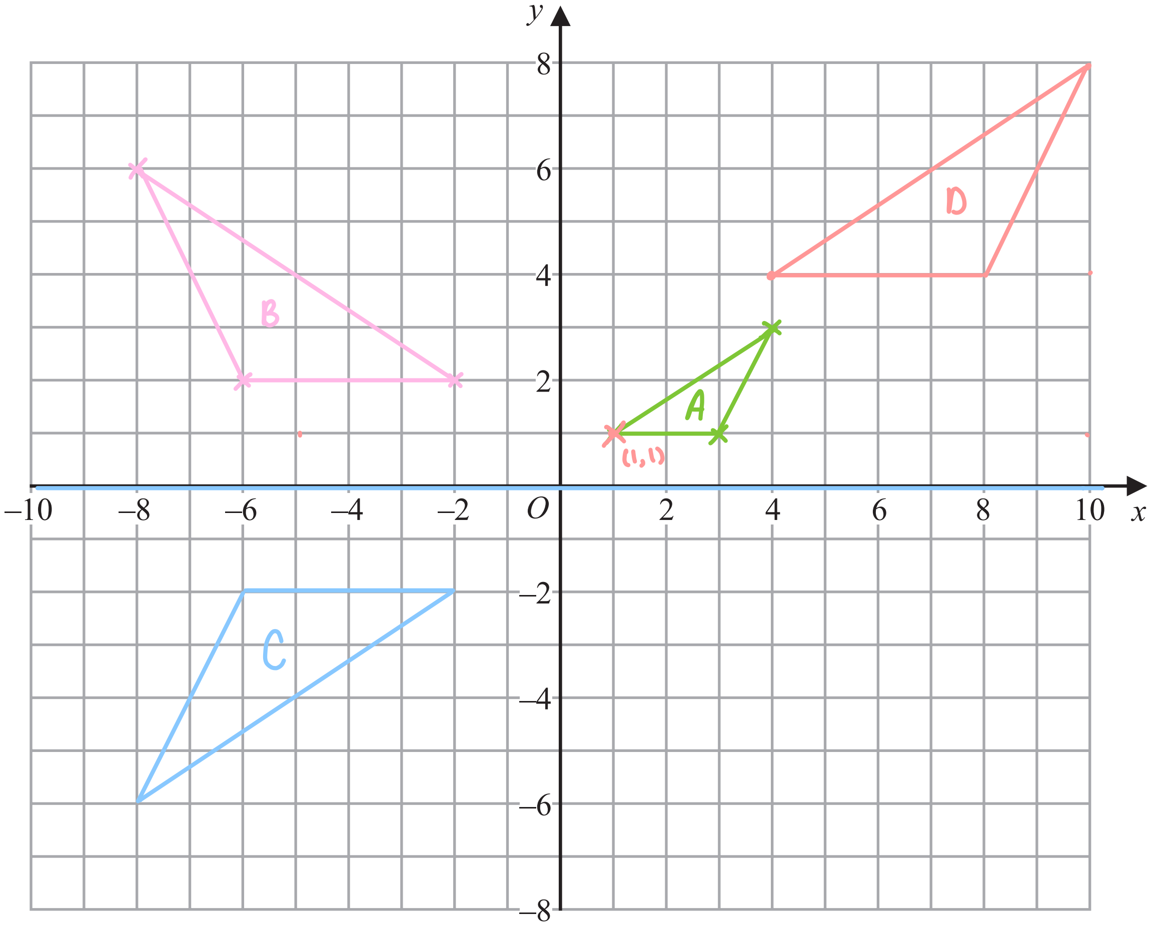
To multiply a 2x2 matrix by a vector:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2(1) + 0(1) \\ 0(1) + 2(1) \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Question 6 continued



$$\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2(3)+0(1) \\ 0(3)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -2(4)+0(3) \\ 0(4)+2(3) \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ 6 \end{bmatrix}$$

Turn over for a spare copy of the grid if you need to redraw your triangles.

Question 6 continued

(c) Draw the horizontal line $y=0$ and reflect the triangle B.

(d) Assume $N = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and work backwards from the coordinates of A and C.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$$

$a(1) + b(1) = -2$ $\qquad$ $(3)a + (1)b = -6$

$a + b = -2$ $\qquad$ $3a + b = -6$

$c(1) + d(1) = -2$ $\qquad$ $(3)c + (1)d = -2$

$c + d = -2$ $\qquad$ $3c + d = -2$

Solve simultaneously for a, b, c and d.

Remember: Same Sign Subtract

$$\begin{aligned} 3a + b &= -6 \\ -\quad a + b &= -2 \\ \hline 2a &= -4 \\ a &= -2 \end{aligned}$$

$\div 2$ $\qquad$ $\div 2$

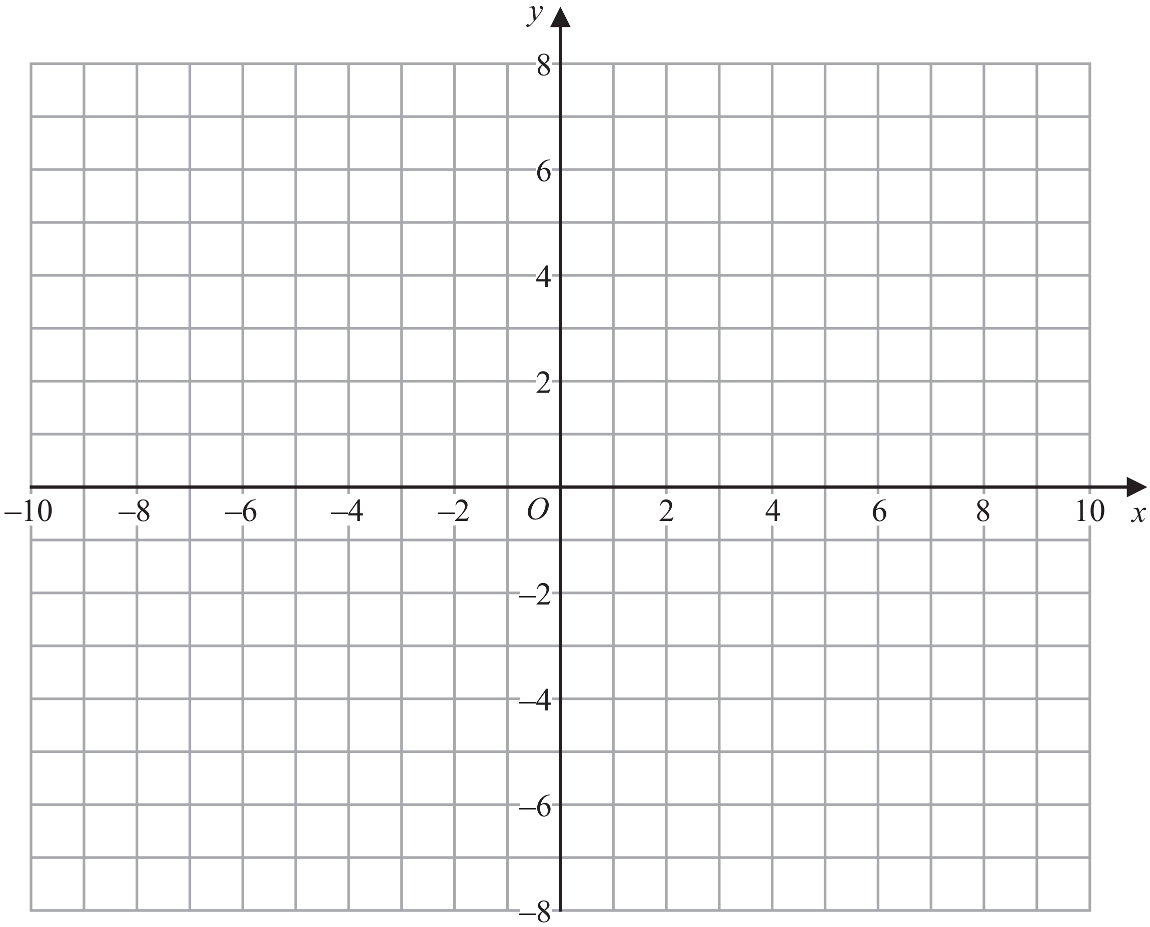
Sub $a = -2$ →

$$\begin{aligned} (-2) + b &= -2 \\ b &= 0 \end{aligned}$$

$+2$ $\qquad$ $+2$

Question 6 continued

Only use this grid if you need to redraw your triangles.



$3c + d = -2$

$-\quad c + d = -2$

$2c = 0$

$\div 2 \qquad c = 0 \qquad \div 2$

Sub $c=0$ $\rightarrow$ $(0) + d = -2$

$d = -2$

$\therefore N = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

(Total for Question 6 is 12 marks)

Question 6 continued

(e) Draw triangle C on tracing paper and mark the point (1,1) with a cross. Rotate the tracing paper 180°, ensuring the cross remains on (1,1) throughout and draw triangle D.

(f) Enlargement of scale factor $\frac{1}{2}$ around centre of enlargement (-2,2)

7 The table gives information about the length of time, in seconds, that each of 28 birds spent on Ali's bird table on Monday.

Length of time (t seconds)	midpoint	**Frequency**	
$0 < t \leqslant 5$	2.5	10	10
$5 < t \leqslant 8$	6.5	8	18 ← The 14th bird is in this interval.
$8 < t \leqslant 10$	9	5	23
$10 < t \leqslant 15$	12.5	3	26
$15 < t \leqslant 30$	22.5	2	28
$t > 30$		0	28

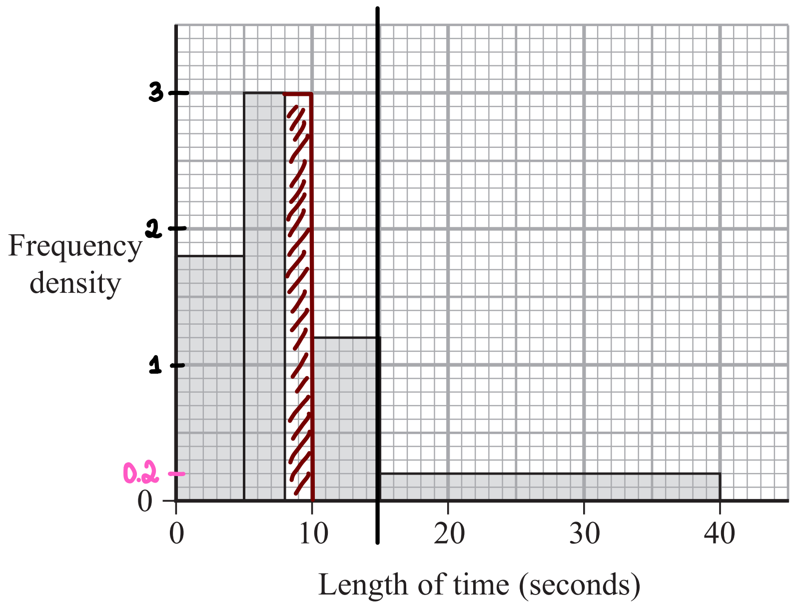
(a) Find the class interval that contains the median length of time that the birds spent on the bird table on Monday.

(1)

(b) Calculate an estimate, in seconds to one decimal place, for the mean length of time that the birds spent on the bird table on Monday.

(4)

The incomplete histogram below gives some information about the length of time, in seconds, that each of 35 birds spent on Ali's bird table on Tuesday.



On Tuesday

9 of the birds spent between 5 seconds and 8 seconds on the bird table
6 of the birds spent between 8 seconds and 10 seconds on the bird table
no bird spent longer than 40 seconds on the bird table.

(c) Use this information to complete the histogram.

(1)

(d) Find the proportion of birds on Tuesday that spent longer than 15 seconds on the bird table.

(1)

Turn over for a spare copy of the histogram if you need to redraw your bar.

Question 7 continued

(a) $28 \div 2 = 14$

$\therefore 5 < t \leq 8$

(b) For the mean multiply the midpoint of each interval by the frequency and

$$\frac{(2.5 \times 10) + (6.5 \times 8) + (9 \times 5) + (12.5 \times 3) + (22.5 \times 2)}{10 \quad 8 + 5 + 3 + 2} = \frac{204.5}{28}$$

$= 7.30357...$

≈ 7.3 (1 d.p.)

(c) you need to find the frequency density for each interval and complete the histogram

$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

$\frac{9}{8-5} = 3$

You can work out the intervals using this

$\frac{6}{10-8} = 3$

(d) You need to find the frequency for the interval 15min to 40min and find the proportion by dividing by the total frequency.

$0.2 \times (40 - 15) = 5$

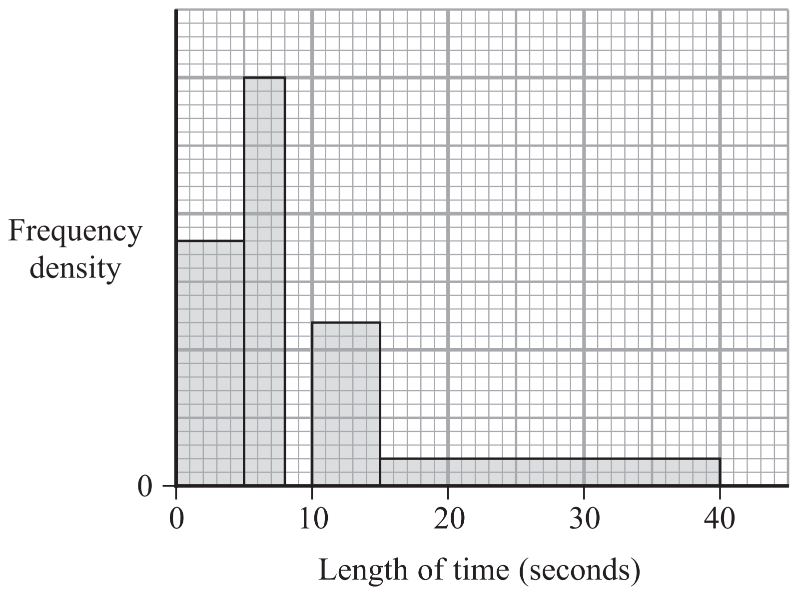
$\therefore \frac{5}{35} \overset{\div 5}{=} \frac{1}{7}$

total number of birds

Question 7 continued

Question 7 continued

Only use this histogram if you need to redraw your bar.



(Total for Question 7 is 7 marks)

8 Jenny ran a road race.

The distance Jenny ran was 5 km, to the nearest 20 m.
Jenny's time for the race was 34 minutes, to the nearest minute.

Colin ran a different road race.

The distance Colin ran was 10 km, to the nearest 200 m.
Colin's time for the race was 1 hour 8 minutes, to the nearest minute.

Colin's average speed for his race is greater than Jenny's average speed for her race.

Calculate the upper bound for the difference, in km/h, between Colin's average speed and Jenny's average speed.
Show your working clearly.

(5)

First you need to write the bounds for both Colin's and Jennie's journeys

Jenny.

$4.99 \text{ km} \leq \text{Jenny's distance} < 5.01 \text{ km}$

$33.5 \text{ min} \leq \text{Jenny's time} < 34.5 \text{ min}$

Colin:

$9.9 \text{ km} \leq \text{Colin's distance} < 10.1 \text{ km}$

$67.5 \text{min} \leq \text{Colin's time} < 68.5 \text{ min}$

For the upper bound in speed between Colin's and Jenny's speeds, you need the largest possible value. So you need the upper bound for Colin's speed and the lower bound for Jenny's speed in km/h.

Speed = distance ÷ time

Upper bound for Colin's speed:
For the upper bound for speed, you need the upper bound for distance and the lower bound for time.

P 6 2 0 1 9 A 0 2 0 3 6

Question 8 continued

First convert time to hours.

1 hour = 60 minutes

$67.5 \div 60 = 1.125$ hours

Upper bound for Colin's speed $= \dfrac{10.1}{1.125} = 8.977...$ km/h

Lower bound for Jenny's speed.

To find the lower bound for speed, you need the lower bound for distance and the upper bound for time.

$34.5 \div 60 = 0.575$ hours

Lower bound for Jenny's speed $= \dfrac{4.99}{0.575} = 8.678...$ km/h

Upper bound for difference in speed $= 8.977... - 8.678...$

$= 0.299516...$

≈ 0.2995 km/h

(Total for Question 8 is 5 marks)

9 Figure 2 shows two shapes A and B.
All the lengths are in metres.

Diagram **NOT** accurately drawn

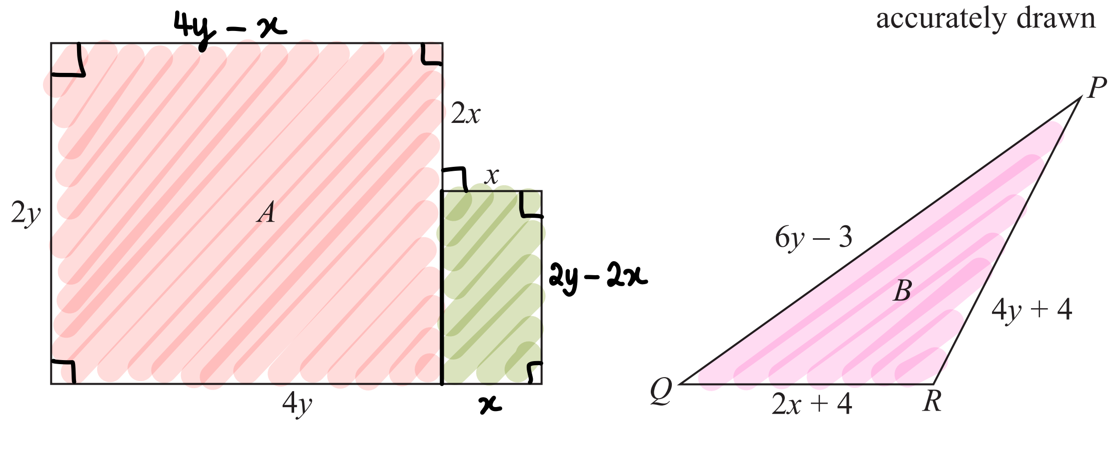


Figure 2

The corners of shape A are all right angles. → You can form 2 rectangles
The area of shape A is $400\,\text{m}^2$
Shape B is triangle PQR.

The perimeter of shape A is equal to the perimeter of shape B.

(a) Show that $4y^2 - x^2 = 200$ and that $2y = 2x + 5$

(3)

(b) Find the value of x and the value of y.
Show clear algebraic working.

(5)

(a) First form an expression for the area of shape A in terms of x and y.

$(2y)(4y-x) = 8y^2 - 2yx$

$(x)(2y-2x) = 2yx - 2x^2$

$(8y^2 - 2yx) + (2yx - 2x^2) = 400$

$8y^2 - 2x^2 = 400$

$\div 2 \quad 4y^2 - x^2 = 200 \quad \div 2$

Then make expressions for the perimeter of A and B and make them equal.

Perimeter of $A = (4y-x) + (2y) + (2x) + (4y) + (x) + (2y-2x)$

$= 12y$

Question 9 continued

Perimeter of B = $(6y-3)+(4y+4)+(2x+4)$

$= 10y+2x+5$

$$12y = 10y+2x+5 \quad -10y$$
$$2y = 2x+5$$

(b) You can use the two expressions from part (a) to solve simultaneously for x and y.

① $4y^2 - x^2 = 200$ ② $2y = 2x+5$

Rearrange equation ② to make y the subject and substitute into equation ①.

$$2y = 2x+5 \quad \div 2$$
$$y = x + \frac{5}{2}$$

	x	$\frac{5}{2}$
x	x^2	$\frac{5x}{2}$
$\frac{5}{2}$	$\frac{5x}{2}$	$\frac{25}{4}$

$x^2 + \frac{5x}{2} + \frac{5x}{2} + \frac{25}{4}$

$= x^2 + \frac{10x}{2} + \frac{25}{4}$

$$4\left(x+\frac{5}{2}\right)^2 + x^2 = 200$$

$$4\left(x^2+\frac{5x}{2}+\frac{25}{4}\right) + x^2 = 200$$

$$4x^2 + 10x + 25 + x^2 = 200$$

$$5x^2 + 10x + 25 = 200$$

$$5x^2 + 10x - 175 = 0$$

$$x^2 + 2x - 35 = 0$$

$$(x+7)(x-5) = 0$$

What two numbers add to make 2 and multiply → to make -35?

7 and -5

Question 9 continued

$$x + 7 = 0 \xrightarrow{-7} x = -7 \qquad x - 5 = 0 \xrightarrow{+5} x = 5$$

Since you cannot have a negative value for length, x must be 5.

Substitute $x = 5$ into equation 2 to find a y value.

$2y = 2(5) + 5$

$2y = 10 + 5$

$2y = 15$

$y = \frac{15}{2}$

$\therefore \quad x = 5 \qquad y = \frac{15}{2}$

Question 9 continued

(Total for Question 9 is 8 marks)

10 $$f(x) = 3x^3 - 7x^2 + 5x - 1$$

(a) Use the factor theorem to show that $(3x - 1)$ is a factor of $f(x)$.

(2)

(b) Hence solve the equation $f(x) = 0$
Show clear algebraic working.

(3)

The curve C has equation $y = f(x)$

(c) Find the exact coordinates of the turning points on C.

(5)

The curve C crosses the y-axis at the point P.

(d) (i) Find the gradient of C at P.

(1)

(ii) Hence find an equation of the tangent to C at P.

(2)

(a) Since $(3x-1)$ is a factor of $f(x)$, when you rearrange $3x-1=0$ to make x the subject and substitute into $f(x)$, $f(x)$ will equal 0.

$3x - 1 = 0$ $\xrightarrow{+1}$ $3x = 1$ $\xrightarrow{\div 3}$ $x = \frac{1}{3}$

$$f\left(\tfrac{1}{3}\right) = 3\left(\tfrac{1}{3}\right)^3 - 7\left(\tfrac{1}{3}\right)^2 + 5\left(\tfrac{1}{3}\right) - 1$$
$$f\left(\tfrac{1}{3}\right) = \frac{3}{27} - \frac{7}{9} + \frac{5}{3} - 1$$
$$f\left(\tfrac{1}{3}\right) = \frac{3}{27} - \frac{21}{27} + \frac{45}{27} - \frac{27}{27}$$
$$f\left(\tfrac{1}{3}\right) = \frac{0}{27}$$
$$f\left(\tfrac{1}{3}\right) = 0$$

(b) You can now use long division and find the remaining factors.

Question 10 continued

$$\begin{array}{r|l} & x^2 - 2x + 1 \\ 3x-1 & 3x^3 - 7x^2 + 5x - 1 \\ & -\underline{(3x^3 - x^2)} \\ & -6x^2 + 5x \\ & -\underline{(-6x^2 + 2x)} \\ & 3x - 1 \\ & -\underline{(3x - 1)} \\ & 0 \end{array}$$

$x^2 - 2x + 1 = (x-1)^2$ → What two numbers add to make -2 and multiply to make 1? -1 and -1.

$3x - 1 = 0$

$x = \frac{1}{3}$

$(x-1)^2 = 0$

$\sqrt{\ }$ $x - 1 = 0$ $\sqrt{\ }$

$+1$ $x = 1$ $+1$

(c) To find the coordinates of the turning points, you first need to differentiate to find $f'(x)$ and then solve for $f'(x) = 0$ to find the points where the gradient is 0.

The general rule for differentiation is to "bring down the power and subtract 1"

$f(x) = 3x^3 - 7x^2 + 5x - 1$

$f'(x) = (3)3x^{3-1} - (2)7x^{2-1} + 5$

$= 9x^2 - 14x + 5$

Question 10 continued

$9x^2 - 14x + 5 = 0$ What two numbers add to make -14 and multiply to make 45?

$9 \times 5 = 45$

$\dfrac{(\cancel{9}x - \cancel{9})(9x - 5)}{\cancel{9}} = 0 \rightarrow -9$ and -5

$(x-1)(9x-5) = 0$

$x - 1 = 0$ $\xrightarrow{+1}$ $x = 1$

$9x - 5 = 0$ $\xrightarrow{+5}$ $9x = 5$ $\xrightarrow{\div 9}$ $\boxed{x = \frac{5}{9}}$

You can now find the y co-ordinates by substituting your x values back into f(x).

$f(1) = 3(1)^3 - 7(1)^2 + 5(1) - 1$

$= 3 - 7 + 5 - 1$

$= 0$

$f\left(\frac{5}{9}\right) = 3\left(\frac{5}{9}\right)^3 - 7\left(\frac{5}{9}\right)^2 + 5\left(\frac{5}{9}\right) - 1$

$= \frac{32}{243}$

$\therefore (1, 0)$ and $\left(\frac{5}{9}, \frac{32}{243}\right)$

Question 10 continued

(d) (i) Curve C crosses the y-axis at the y-intercept. The y-intercept is the constant value in the equation for f(x).

y-intercept $\rightarrow$ $(0, -1)$

$f'(0) = 9(0)^2 - 14(0) + 5$

$= 5$

(ii) The tangent has the same gradient as the gradient at P

$\therefore\ m_T = 5$

The tangent has a point with coordinates $(0, -1)$ [P].

$y = mx + c$

$(-1) = 5(0) + c$

$-1 = c$

$\therefore\ y = 5x - 1$

(Total for Question 10 is 13 marks)

11

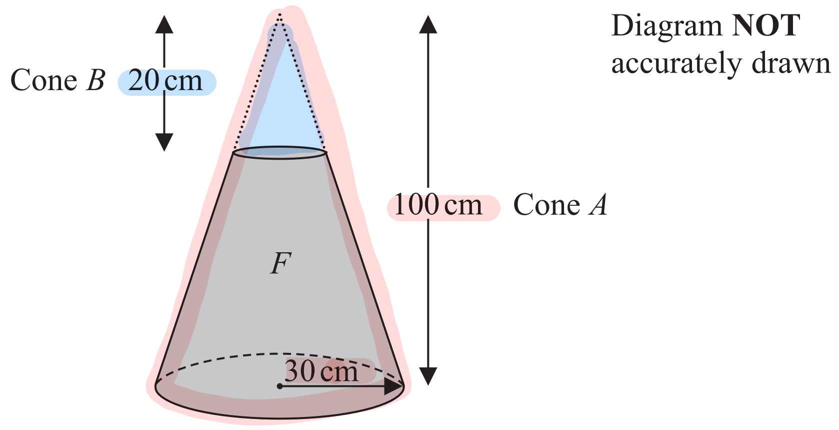


Figure 3

A right circular cone, A, has height 100 cm and base radius 30 cm.
The shaded solid, F, is formed by removing the right circular cone, B, of height 20 cm from the top of A, as shown in Figure 3

The volume of F is $k\pi$ cm^3 where k is an integer.

(a) Find the value of k.

(3)

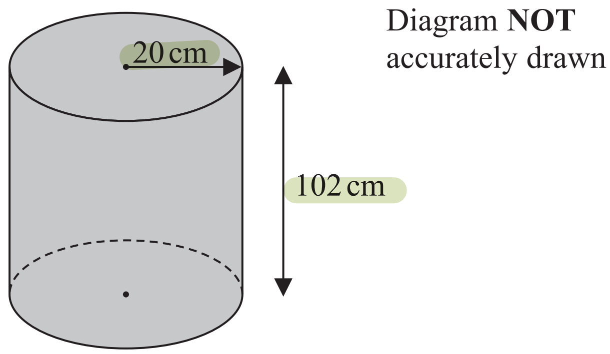


Figure 4

Figure 4 shows a solid circular cylinder.
The cylinder has a radius of 20 cm, a height of 102 cm and a volume of V cm^3

(b) Calculate the value of V.
Give your answer in terms of π.

(1)

$$\left[\begin{array}{ll} \text{Volume of a cylinder} & = \pi r^2 h \\ \text{Volume of a cone} & = \frac{1}{3}\pi r^2 h \end{array}\right]$$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 2 0 1 9 A 0 3 0 3 6

Question 11 continued

(a) You can find the volume of F by subtracting the volume of B from the volume of A.

First find the volume of A using the equation given for the volume of a cone:

Volume of A: $\frac{1}{3}\pi(30)^2(100)$

$= 3000\pi$

You can use the scale factor to find the volume of B:

cube scale factor

square scale factor

Height / Length

Area

Volume

square root scale factor

cube root scale factor

	A	:	B
height	100	$\xrightarrow{\times \frac{1}{5}}$	20
volume	3000π	$\xrightarrow{\times \left(\frac{1}{5}\right)^3}$	600π

Volume of F: $3000\pi - 600\pi$

$= 2400\pi$

$\therefore \underline{\underline{k = 2400}}$

(b) Find the volume V using the equation for volume of a cylinder:

$V = \pi \times (20)^2 \times (102)$

$= 40800\pi$

Question 11 continues on the next page.

Question 11 continued

Figure 5 below shows a door stop made from a hollow hemisphere and a hollow right circular cone.

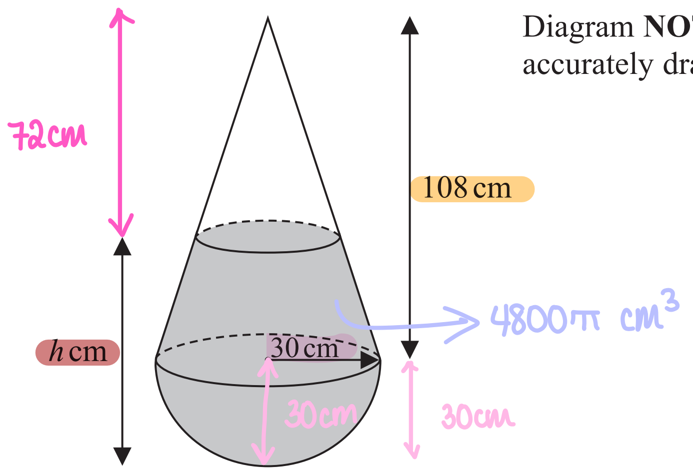


Figure 5

The radius of the base of the cone and the radius of the hemisphere are both 30 cm.
The centre of the base of the cone coincides with the centre of the hemisphere.
The height of the cone is 108 cm.

The door stop rests on horizontal ground with the cone on top of the hemisphere and the axis of symmetry of the door stop vertical.

The door stop contains sand to a height of h cm.

Given that the volume of sand in the door stop is V cm³, where V cm³ is the volume of the cylinder in Figure 4

(c) calculate the value of h.

(6)

(c) We know from part (b) that $V = 40800\pi$.

Since the bottom is a sphere, its height must be the same as the radius so is 30cm and you only need to find the height of the lower part of the cone to find h.

You can first find the volume of the sphere and subtract from V.

Volume of a sphere $= \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi(30)^3 \times \frac{1}{2} = 18000\pi$

$40800\pi - 18000\pi = 22800\pi$

$$\left[\text{Volume of a sphere} = \frac{4}{3}\pi r^3\right]$$

Question 11 continued

You can now find the volume of the smaller cone and the height to find h.

First find the volume of the large cone:

Larger cone: $\frac{1}{3}\pi(30)^2(108) = 32400\pi$

Volume of small cone: $32400\pi - 22800\pi$

$= 9600\pi$

	Large Cone	:	Small Cone
Volume	32400π	:	9600π
			$\times \frac{8}{27}$
Height	108	:	72
			$\times \sqrt[3]{\frac{8}{27}}$

$h = (108 + 30) - 72$

$= \underline{\underline{66 \text{ cm}}}$

(Total for Question 11 is 10 marks)

12

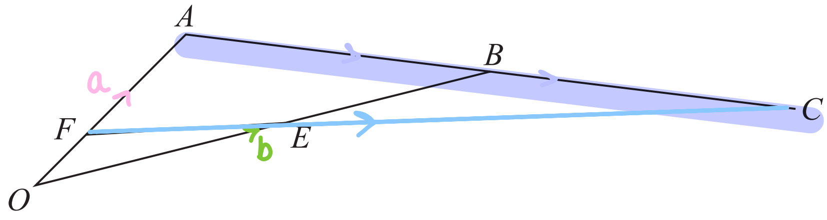


Diagram **NOT** accurately drawn

Figure 6

Figure 6 shows triangle *OAB*.

The point *E* lies on *OB* such that $OE : OB = 1 : 2$

The point *F* lies on *OA* such that $\overrightarrow{OF} = \frac{1}{5}\overrightarrow{OA}$

Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) find $\overrightarrow{FE}$ in terms of **a** and **b**

(2)

The point *C* is such that *ABC* is a straight line and $AB = BC$.

(b) Show that *F*, *E* and *C* are not collinear.

(4)

Given that *ABG* and *FEG* are straight lines,

(c) find $\overrightarrow{OG}$ in terms of **a** and **b**

(5)

(a) $\vec{FE} = \vec{FO} + \vec{OE}$

$\vec{OF} = \frac{1}{5}\vec{OA}$ $\qquad$ $\vec{OE} : \vec{OB}$

$\vec{OA} = a$ $\qquad$ $1 : 2$

$\div 2$

$\vec{OF} = \frac{1}{5}a$ $\qquad$ $\frac{1}{2}b : b$

$\div 2$

$\vec{FO} = -\frac{1}{5}a$

$\underline{\underline{\vec{FE} = -\frac{1}{5}a + \frac{1}{2}b}}$

Question 12 continued

(b) To do this, you must find an expression for the vector $\overrightarrow{FC}$ and show that it is not a multiple of the vector $\overrightarrow{FE}$.

Since $\overrightarrow{AB} = \overrightarrow{BC}$, $\overrightarrow{AC} = 2\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -a + b$

$\overrightarrow{FE} = \overrightarrow{FA} + 2\overrightarrow{AB}$

Since $\overrightarrow{OF} = \frac{1}{5}a$, $\overrightarrow{FA} = \frac{4}{5}a$

$\overrightarrow{FE} = \frac{4}{5}a + 2(b - a)$

$= \frac{4}{5}a + 2b - 2a$

$= 2b - \frac{6}{5}a$

$2b - \frac{6}{5}a$ is not a multiple of $-\frac{1}{5}a + \frac{1}{2}b$ so F, E and C are not collinear.

(c) First find two expressions for $\overrightarrow{OG}$.

$\overrightarrow{OG} = \overrightarrow{OA} + m\overrightarrow{AB}$ $\qquad$ $\overrightarrow{OG} = \overrightarrow{OF} + n\overrightarrow{FE}$

$\overrightarrow{OG} = a + m(b - a)$ $\qquad$ $\overrightarrow{OG} = \frac{1}{5}a + n\left(-\frac{1}{5}a + \frac{1}{2}b\right)$

$\overrightarrow{OG} = a + mb - ma$ $\qquad$ $\overrightarrow{OG} = \frac{1}{5}a - \frac{1}{5}an + \frac{1}{2}bn$

Now separate the a and b terms.

Take out a factor of "a".

$a - ma = \frac{1}{5}a - \frac{1}{5}an$

$\cancel{a}(1 - m) = \cancel{a}\left(\frac{1}{5} - \frac{1}{5}n\right)$

$1 - m = \frac{1}{5} - \frac{1}{5}n$

Question 12 continued

Take out a factor of "b"

$mb = \frac{1}{2}bn$

$\cancel{b}(m) = \cancel{b}\left(\frac{1}{2}n\right)$

$m = \frac{1}{2}n$

$1 - \left(\frac{1}{2}n\right) = \frac{1}{5} - \frac{1}{5}n$

$-\frac{1}{5}$ $\quad$ $-\frac{1}{5}$

$\frac{4}{5} - \frac{1}{2}n = -\frac{1}{5}n$

$+\frac{1}{2}n$ $\quad$ $+\frac{1}{2}n$

$\frac{4}{5} = \frac{3}{10}n$

$\div\frac{3}{10}$ $\quad$ $\div\frac{3}{10}$

$\frac{8}{3} = n$

$m = \frac{1}{2}\left(\frac{8}{3}\right)$ $\leftarrow$ Substitute $n = \frac{8}{3}$ into $m = \frac{1}{2}n$

$m = \frac{4}{3}$

$\vec{OG} = a + \frac{4}{3}(b - a)$

$= a + \frac{4}{3}b - \frac{4}{3}a$

$= \frac{4}{3}a - \frac{1}{3}b$

(Total for Question 12 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS